Preston & Wingham Primary Schools Federation

Learning together, we grow kind hearts and healthy minds.

Progression in the teaching of calculation

This document has been produced in line with the Primary National Curriculum for mathematics and the White Rose *Maths*HUBS Calculation Policy in order to help you in supporting your child in mathematics.

$$5 + 1 = 6$$
$$5 + 2 = 7$$
$$5 + 3 = ?$$

Here at the Preston and Wingham Primary Schools Federation, we believe that in order for children to become confident, curious and resilient mathematicians, maths should be fun and engaging for learners of all ages by creating as many opportunities as possible to make links not only with other areas of maths and the wider school curriculum, but also in learners' everyday experiences away from school. This might, for example, be while playing at home with their toys and sorting or counting objects; going to the shops with a grown up and spending money; or looking at shapes and patterns both in nature and in manmade structures in the town centre. In this way, we hope that the use of mathematics will gradually become second nature to the children and will form part of their daily experiences in the same way that speaking and listening does.

By teaching in stages and showing them a variety of ways to calculate, we encourage children to identify patterns and make connections so we can ensure that they really understand what they are doing rather than just following a set of instructions. As part of our teaching, we use the idea of 'variation' in both a child's conceptual, and procedural, understanding. The use of both 'standard' and 'non-

standard' examples is a key feature of this approach as this enables children to fully understand the features of something like, say, a triangle. The use of '*non-examples*' is also a key feature of the teaching of mathematics as it helps children of all ages identify *why* something *is*, say, a triangle and why something else is not thus developing, and promoting use of, learners' reasoning skills which in turn has a direct impact on their problem solving ability.

What you need to know about calculations

Mathematics will be at the core of your child's schooling from the moment they start to the moment they leave. They will be involved in drawing, measuring, handling data and lots of other practical activities that will help your child to understand and enjoy the subject. The methods we are advocating follow the White Rose *Maths*HUBS programme of study and are in line with the National Curriculum (September 2014).

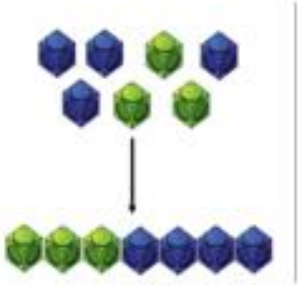
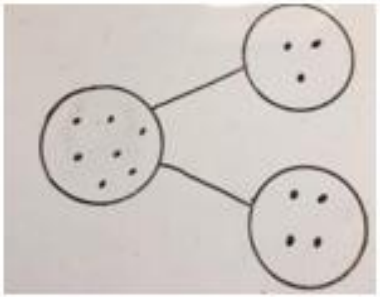
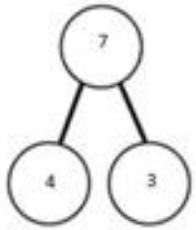
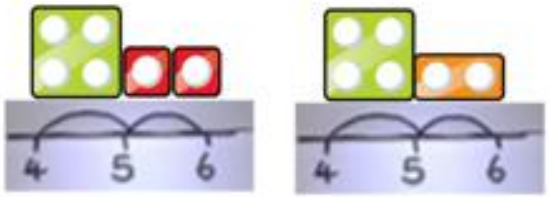
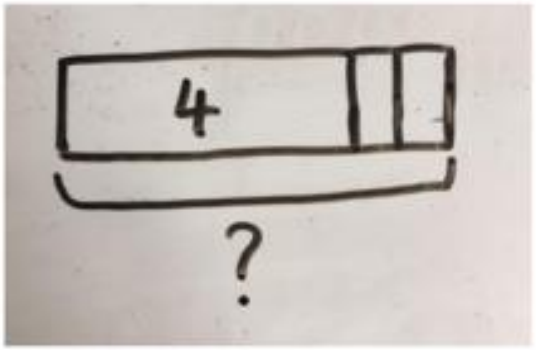
The methods we use in school may or may not be familiar to you. Children are often confused when they ask parents for help at home and parents try to teach them the methods that *they* themselves were once taught. Knowing how the methods in this document work will help you to help your child. All staff members in school work from this document so that we can ensure consistency of our approach and can make sure that the children move onto the next step *when they are ready*.

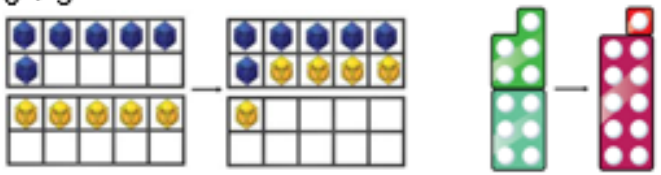
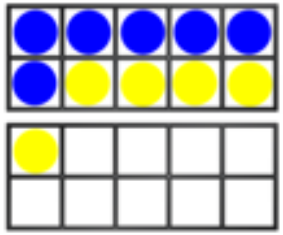
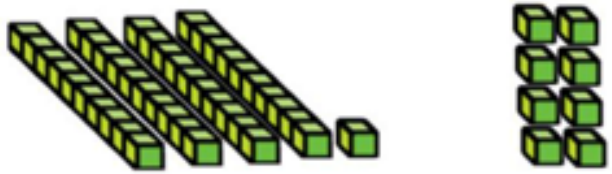
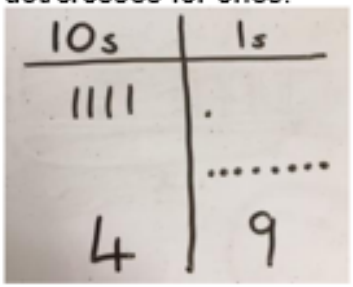
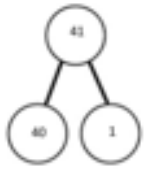
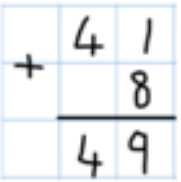
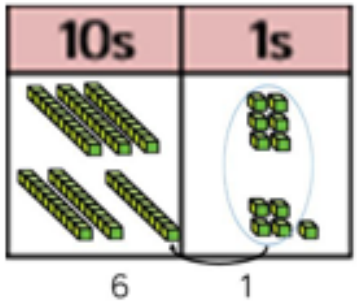
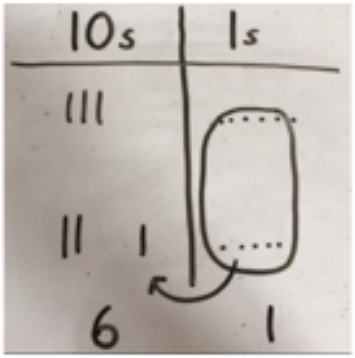
The 4 operations covered in this leaflet are addition, subtraction, multiplication and division. Whichever operation is being taught, the child needs to experience all of the following steps (known as CPA) to fully master it:

1) Concrete – using *physical* objects/resources such as counters, Base 10, Numicon, etc. to represent numbers
2) Pictorial – drawing a picture/diagram which represents the physical resources previously used to represent numbers
3) Abstract – solving problems using only numbers

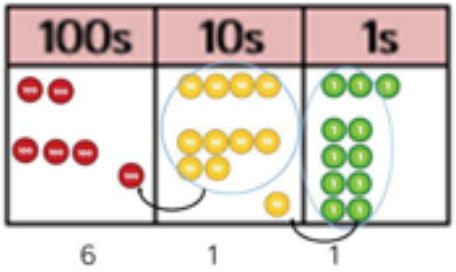
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

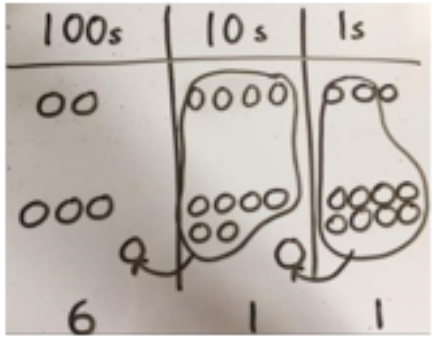
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4+3=7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 +□=11 6 +5=5+□ 6 +5=□+4
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41+8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41+8  1+8=9 40+9=49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36+25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10. 36 + 25=  30+20=50 5+5=10 50+10+1=61 36 Formal method: +25 61 1

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

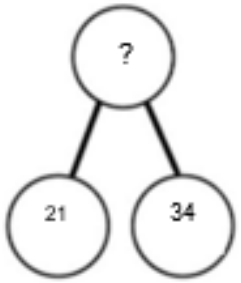


Chidren to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

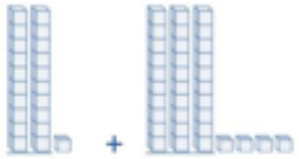
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21+34=

$\square$ =21+34

Calculate the sum of twenty-one and thirty-four.

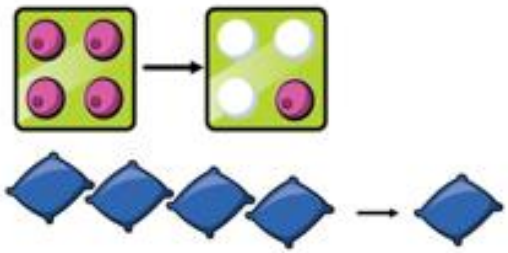
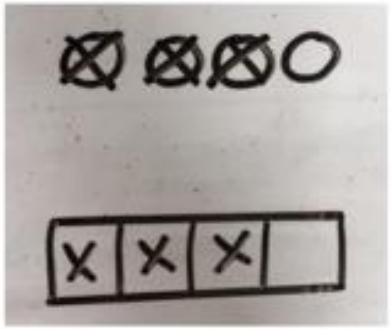
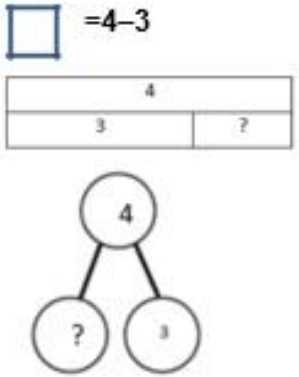
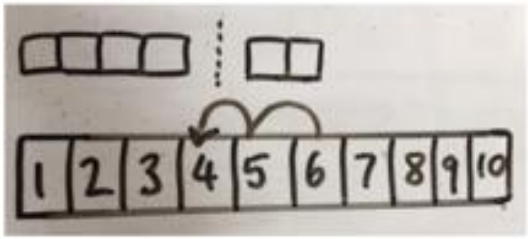
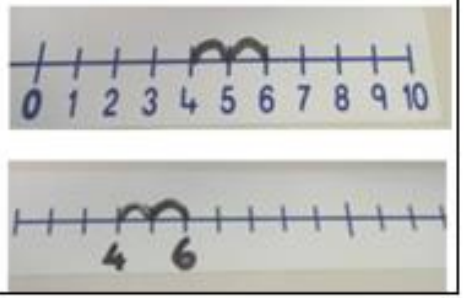


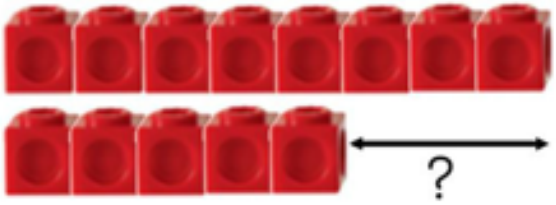
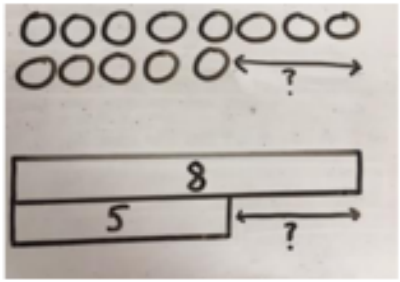
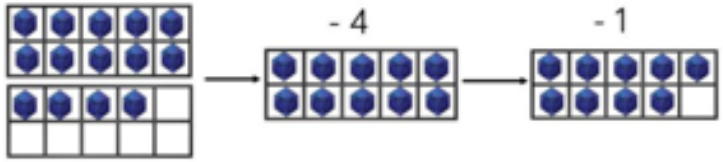
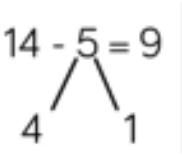
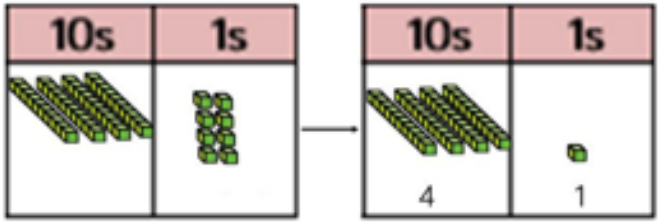
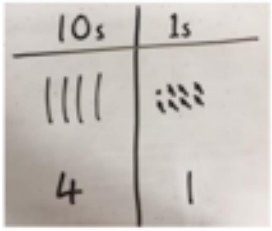
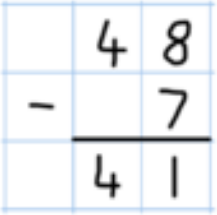
Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

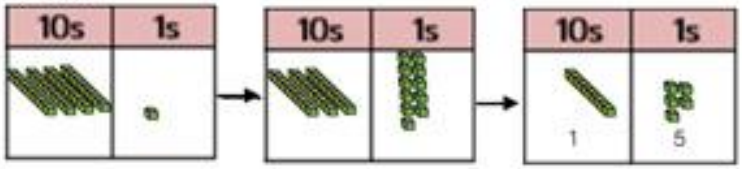
Calculation policy: subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4–3=1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4-3= 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6–2=4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

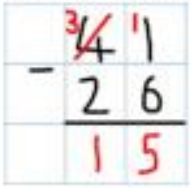
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4. Why do they all have the same difference?
Making 10 using ten frames. 14–5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14–4=10 10–1=9
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 

Column method using base 10 and having to exchange.
41–26

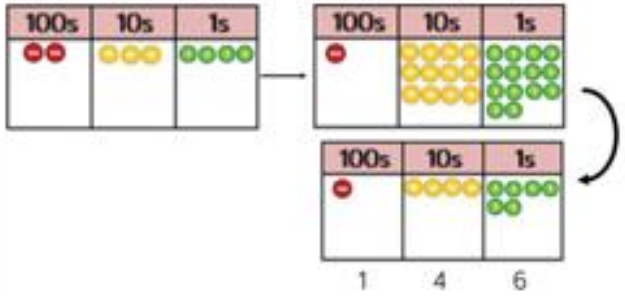


Represent the base 10 pictorially, remembering to show the exchange.

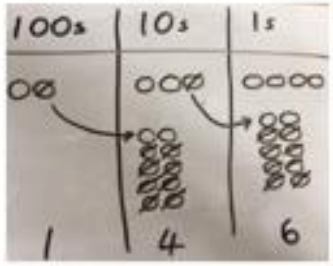
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.



Column method using place value counters.
234 – 88



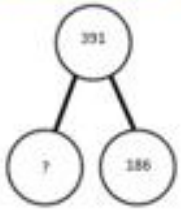
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal colum method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} {}^{2}\cancel{3}^{1}4 \\ -\ 88 \\ \hline 6 \end{array}$$

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
———

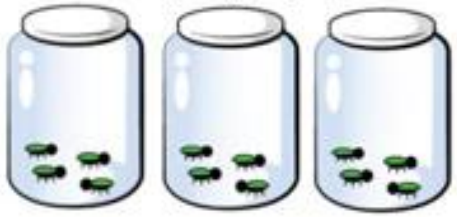
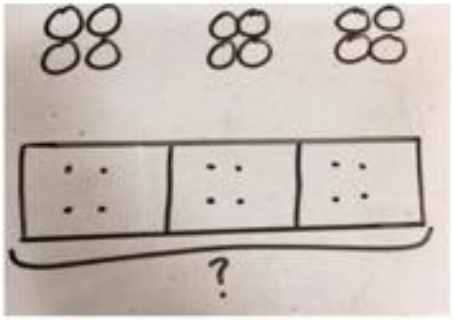
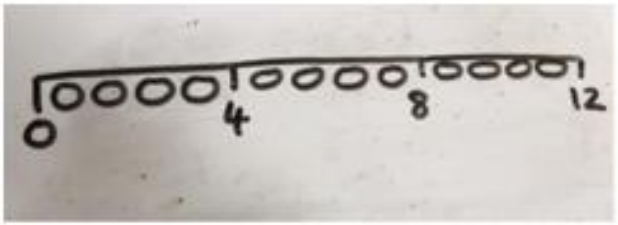
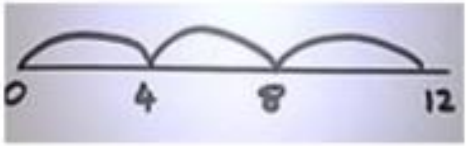
What is 186 less than 391?

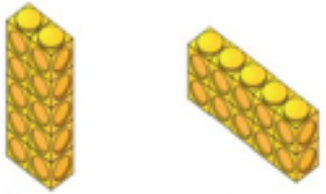
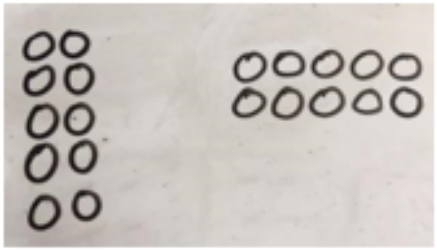
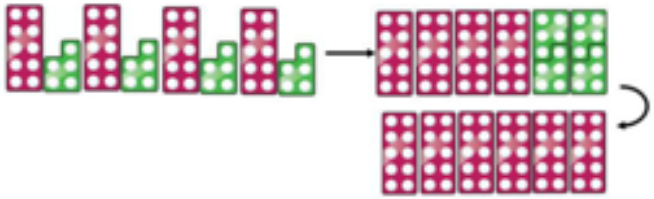
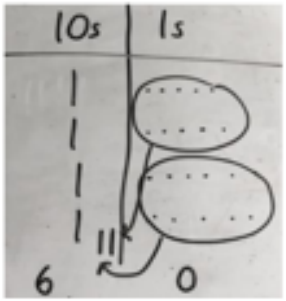
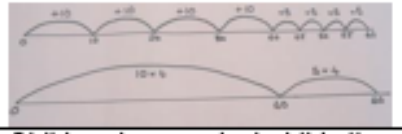
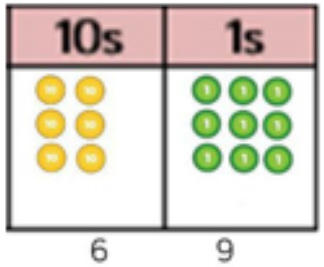
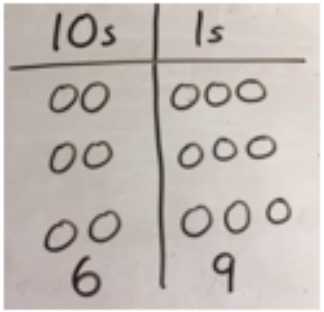
Missing digit calculations

$$\begin{array}{r} 3\ 9\ \square \\ -\ \square\ \square\ 6 \\ \hline \square\ 0\ 5 \end{array}$$

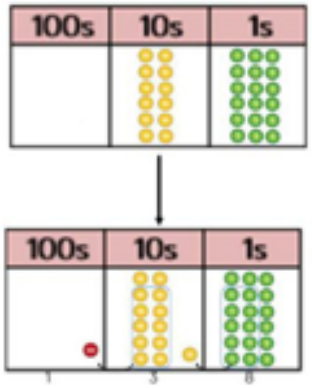
Calculation policy: multiplication

Key language: double, times, multiplied by, the product of, groups of, equal groups.

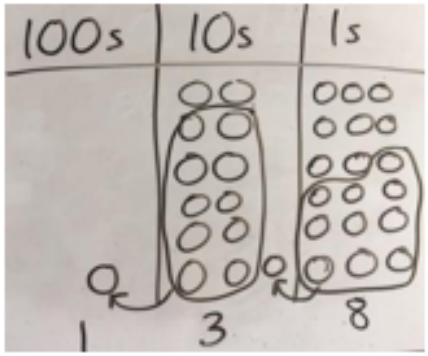
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 4 x 3 4+4+4 There are 3 equal groups, with 4 in each group.   **Number lines to show repeated groups**- 4 x 3  Cuisenaire rods can be used too.	Children to represent the practical resources in a picture and use a bar model.  Represent this pictorially alongside a number line e.g.: 	4 x 3 =12 4+4+4=12 Abstract number line showing three jumps of four. 4 x 3 = 12 

Use arrays to illustrate commutativity. Counters and other objects can also be used. 2×5=5×2  5 groups of 2 2 groups of 5	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 = 2+2+2+2+2 = 10 10 = 5 x 2 = 5+5 = 10
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 15 x 4 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 15 x 4 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (Base 10 can also be used.) 23 x 3 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 23x3 20 x 3 =60 3×3= 9 20 3 60+9=69 23 × 3 69

Formal column method with place value counters.
23



Children to represent the counters/base 10, pictorially
E.g. the image below.



Formal written method

$$23 \times 6 =$$

$$\begin{array}{r} 23 \\ \times\ \ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3 digits × 3 digits and 4 digits × 2 digits etc., they should be confident with the abstract:

To get 744 children have solved 124 x 6.
To get 2480 they have solved 124 x 20.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 23 x 6

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 23 x 6 = 138

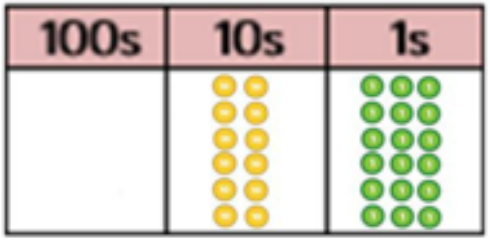
Find the product of 6 and 23

6×23=

□ = 6 × 23

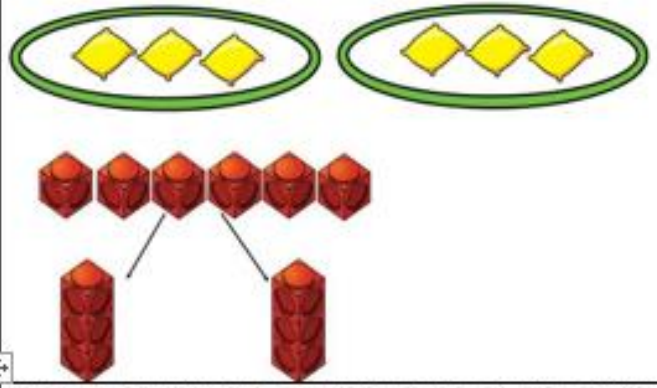
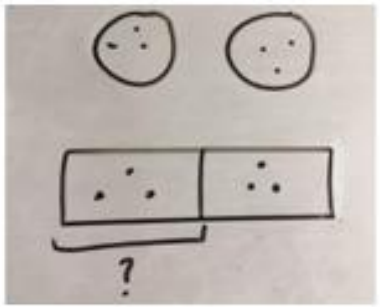
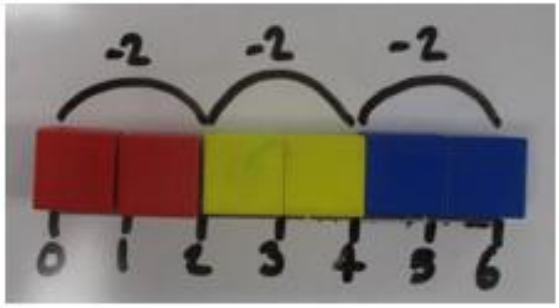
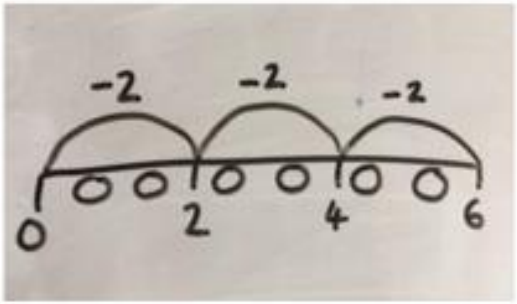
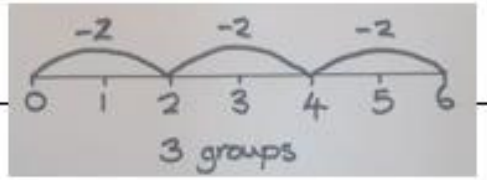
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ \ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?



Calculation policy: Division

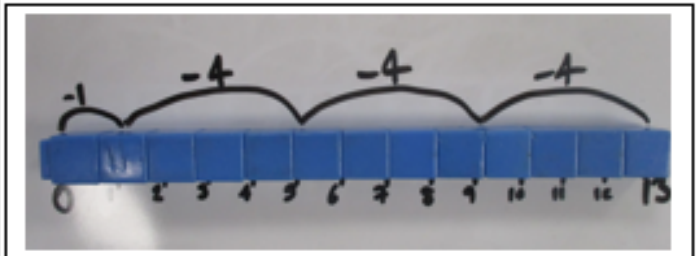
Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6÷2=3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction e.g. using cubes above a number line. 6 ÷ ☐ = 2 Children may decide to reorder the calculation so that it is written as 6 ÷ 2 = ? 	Children to represent repeated subtraction 	Abstract number line to represent the equal groups that have been subtracted. 

2 digits ÷ 1 digit with remainders e.g. using cubes.

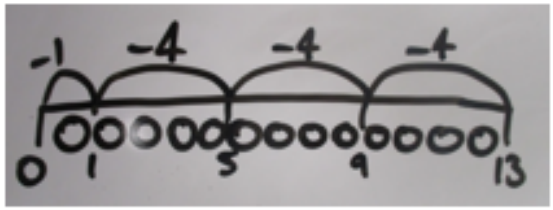
13 ÷ ☐ = 4 r? becomes 13 ÷ 4

Children should be encouraged to apply their knowledge of number to recognise that there will be a remainder.



There are 3 groups of 4, with 1 left over.

Children to represent repeated subtraction to show remainder.

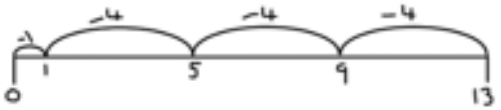


There are 3 groups of 4, with 1 left over.

13 ÷ ☐ = 4 r?

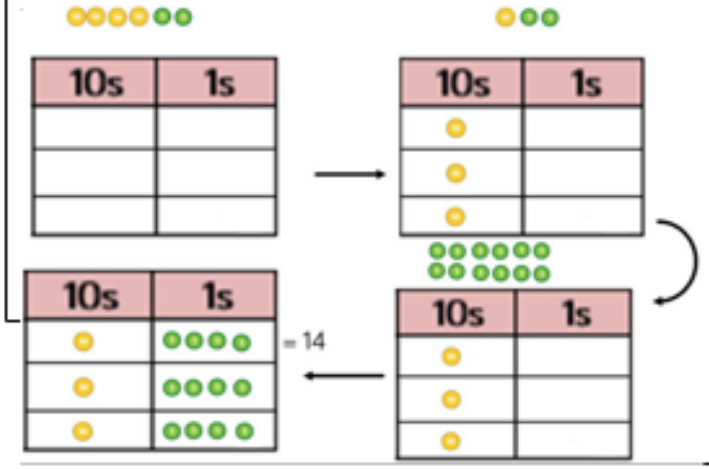
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

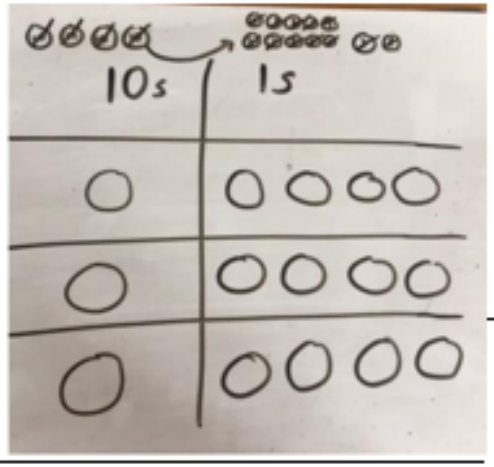


Sharing using place value counters.

42÷3=14



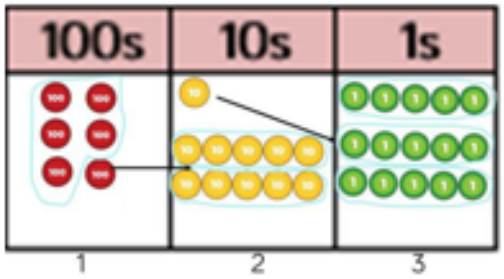
Children to represent the place value counters pictorially



Children to be able to make sense of the place value counters and write calculations to show the process.

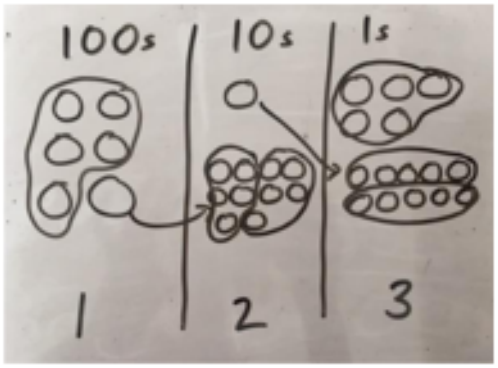
42÷3
42=30+12
30÷3=10
12÷3=4
10+4=14

Short division using place value counters to group.
615÷5



1.Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

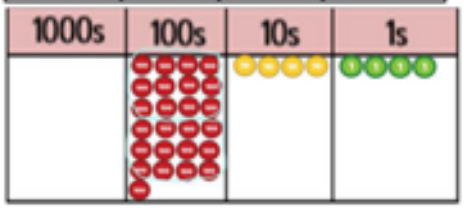


Children do the calculation using the short division scaffold.

$$5\overline{)615} = 123$$

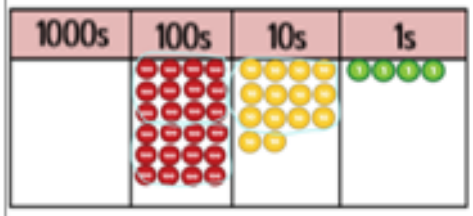
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



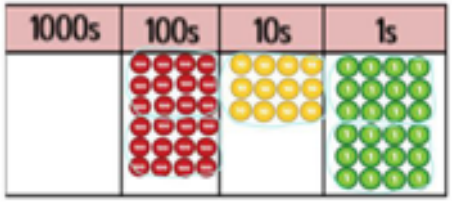
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$5\overline{)615}$

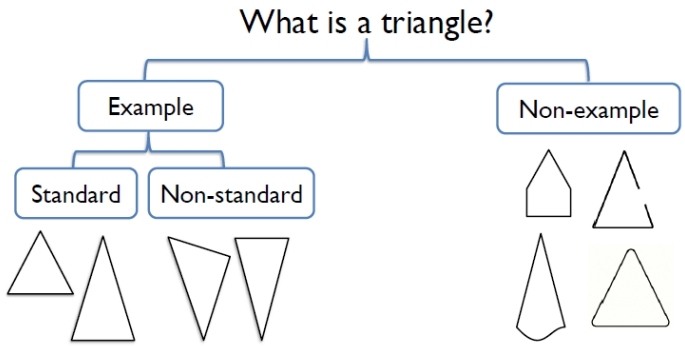
615÷5=

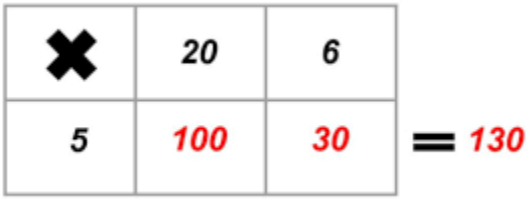
□ =615÷5

What is the calculation?
What is the answer?

Glossary

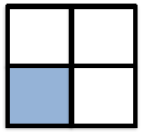
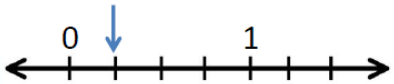
Term	Definition	Examples
2 digit	a number made up of 2 digits	23, 48, 59, 12
3 digit	a number made up of 3 digits	213, 504, 897
Addend	the numbers being added together to form a **sum**	51 + 182
Array	an arrangement of a set of numbers or objects in rows and columns	
Bridge to ten	a strategy when using number lines. Adding a number that takes you to the next tens number	
Bus Stop Method	a traditional method for division with a single digit divisor	
Column Chunking	method of division involving taking chunks or groups or the divisor away from the larger number	
Commutative law	addition and multiplication are both 'commutative'. With addition, numbers can be added in any order and the result will always be the same. The same applies for multiplication.	Addition: 6 + 3 + 5 = 5 + 6 + 3 Multiplication: 2 x 4 x 3 = 4 x 3 x 2 = 3 x 4 x 2

Conceptual variation	use of examples and 'non-examples' (e.g. a triangle) to help learners understand what is correct and what is not; use of standard and non-standard examples (e.g. pictorial representations of *a quarter*)	
Decimal number	a number with a decimal point	3.8, 52.04, 9.866
Difference	the amount by which one number or quantity is more or less than another. It is found by subtracting one quantity from the other	
Dividend	the number being split (or divided) up	12 ÷ 6 where 12 is the dividend
Divisor	the amount of groups the **dividend** is being split into to leave a **quotient**	12 ÷ 6 = 2 where 6 is the divisor
Double	multiply a number by 2	Double 6 = 6 x 2
Exchanging	Moving a 'ten' or 'hundred' from its column into the next column and splitting it up into ten 'ones' (units) or ten 'tens' and putting into a different column	
Find the difference	A method for subtraction involving counting up from the smaller to the larger number	

Grid method	a method for multiplying two numbers together involving partitioning	26 x 5 
Half	a number or shape or quantity that is one of two equal parts of a whole	
Halve	to divide a number by 2 (to split it into <u>two equal parts</u>)	To halve 18 = 18 ÷ 2
Integer	a number with no decimal point	8, 2046, 39
Inverse	the opposite operation. Addition is the inverse of subtraction; multiplication is the inverse of division.	18 + 4 = 22, so 22 – 4 = 18 It is commonly used in missing number problems: 15 + ☐ = 22 can be solved by doing 22 – 15 = ☐ Here, the addition has been rewritten as a subtraction so the **addend** has become a **subtrahend** to be subtracted from the **sum** in order to find the missing number
Long multiplication	column multiplication where only the significant figures are noted	
Multiplicand	the number that is being multiplied - could also be referred to as the amount <u>in</u> a group	'3 groups of 7' 7 is the multiplicand and 3 is the **multiplier** (7 x 3)

Multiplier	the number by which another number (multiplicand) is multiplied	6 x 8 where a group of 6 is being multiplied by 8 (6 is the multiplicand; 8 is the multiplier)
Numberline chunking	method of division that involves taking chunks of the divisor away from the larger number (or dividend)	
Number bonds to 10	2 numbers that add together to make ten, like 3 and 7, 8 and 2	
Number bonds to 100	2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88	
Number sentences	writing out a calculation with just the numbers in a line	2 + 4 = 6 21 ÷ 3 = 7
Operation	a mathematical process combining numbers through either + (addition), - (subtraction), x (multiplication) or ÷ (division)	18 – 10 = 8 Here the subtraction operation is being used
Partition	to split up numbers with two or more digits into their constituent parts or even to split up smaller 1 digit numbers to help with certain calculations such as subtractions using the 'part part whole' model	328 = 300 and 20 and 8 Part part whole example: 14 – 5 where 5 is split into 4 and 1 to become 14 – 4 = 10; then 10 – 1 = 9

Place value	knowing that in the number 342 the 3 means three hundreds, the 4 means four tens and the 2 means two units	
Procedural variation	making changes within a procedure leading to a different result in order to be able to identify patterns and make connections	5+1=6 5+2=7 5+3=8 the second **addend** each time increases by 1, therefore the **sum** each time increases by 1 9 – 5 = 8 – 5 = 7 - 5 = What's stayed the same? What is different?

Quarter	a number or shape or quantity that is one of four equal parts of a whole	**What does $\frac{1}{4}$ mean?** One quarter of a whole  One quarter of a quantity  A number on a number line  One divided by four $1 \div 4 = ?$
Quotient	the amount in each group after a **dividend** is divided by a **divisor**	'12 ÷ 6 = 2' where 2 is the quotient. 12 has been split into 6 equal groups with **2 in each group**
Recombine	for addition, once you have partitioned numbers and added them separately you then add the answers together for the total	62 + 34 becomes 60 + 30 = 90 2 + 4 = 6 Then 90 + 6 = 96
Remainder	a while number left over after division	31 ÷ 4 = 7 **r3** We can only count in 4s seven times, as far as 28, without going past 31. The difference between 28 and 31 is 3 which is left over and is therefore the remainder
Repeated addition	repeated adding groups of the same size for multiplication	3 x 8 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3

Subtrahend	a number being subtracted from another	12 – 9 = 3, where 9 is the subtrahend
Sum	the result of two or more numbers (or **addends**) being added together	24 + 15 = 39, where 39 is the sum

Respect Inclusive Creative Hardworking